AI-1549

M. A. / M. Sc. (Final)

Term End Examination, 2020-21

MATHEMATICS

Paper : Fifth

(Difference Equation)

Time Allowed : Three hours

Maximum Marks : 100

Minimum Pass Marks : 36

Note : Answer any ***five*** *questions. All questions carry equal marks.*

1. (a) Express $f(x) = 3x^4 - 4x^3 + 6x^2 + 2x + 1$ terms of factorial notations.

(b) Prove that :

$$u_0 + \frac{u_1 x}{1!} + \frac{u_2 x^2}{2!} + \frac{u_3 x^3}{3!} + \ldots\ldots\ldots =$$
$$e^x \left[u_0 + x \Delta u_0 + \frac{x^2}{2!} \Delta^2 u_0 + \ldots\ldots\ldots \right]$$

2. (a) Define generating functions, solve :

$y_{x+1} = -y_x + 1, x = 0, 1, 2, 3, \ldots\ldots$, when $y_0 = 1$

(b) Obtain solution of the difference equation of the type :

$$y_{x+1} = A y_x + B$$

3. (a) Solve :

$$y_{x+4} - 4y_{x+3} + 8y_{x+2} - 8y_{x+1} + 4y_x = 0$$

(b) Solve :

$$y_{x+2} - 4y_{x+1} + 4y_x = 3x + 2^x$$

4. (a) Find the Z-transform of $\cosh \frac{n\pi}{2} + \alpha$

(b) If $Z[\langle f(n) \rangle] = F(z)$, then prove that :

$$Z = \left[\left\langle \sum_{k=-\infty}^{n} f(k) \right\rangle \right] = \frac{F(z)}{1 - z^{-1}}$$

5. (a) Solve the difference equation

$$f_{n+3} - 3f_{n+2} + 3f_{n+1} - f_n = F(n)$$

with $f(0) = f(1) = f(2) = 0$ by Z-transforms.

(b) Determine the type and stability of the critical point (0, 0) of the almost linear system

$$\frac{dx}{dt} = 4x + 2y + 2x^2 - 3y^2$$

$$\frac{dy}{dt} = 4x - 3y + 7xy$$

Also, find the general solution of the corresponding linear system.

6. (a) Define self-adjoint. State and prove the necesary and sufficient condition for the second order linear differential equation

$$a_0(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_2(x)y = 0$$

where $a_0(x) \neq 0$ and $a_0''(x)$, $a_1'(x)$ and $a_2(x)$ are continuous on $a \leq x \leq b$.

(b) Find the external of functional $I[y(x)]$ where

$$I[y(x)] = \int_0^{\log 2} (e^{-x} y'^2 - e^x y^2)\, dx$$

7. (a) Obtain non-trival solution of the Sturm-Liouville problem

$$\frac{d^2y}{dx^2} + \lambda y = 0 \text{ with } y(0) = 0,\ y\left(\frac{\pi}{2}\right) = 0$$

(b) Show that the shortest curve joining two fixed points is a straight line.

8. (a) If $[\langle f(n) \rangle] = F(z)$ where

$$F(z) = \frac{2z^2 + 5z + 14}{(z-1)^4}$$

evaluate the value of f_2 and f_3.

(b) State and prove the sturn separation theorem.

9. (a) If the second order difference equation is written in the form of $a_{n+2} - B a_{n+1} + C a_n = 0$, where B and C are constants, show that the stability condition may be written as

$$-1 - C < B < 1 + C,\ C < 1$$

(b) Obtain the necesary condition for

$$\int_{x_1}^{x_2} f(x, y, y')\, dx$$

to be an extremum.

10. (a) Find the plane curve of fixed perimeter and maximum area.

(b) Explain boundary value problems for differential equations.